Anatomical structures determining blood flow in the heart left ventricle

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The presence of twisted helical flow patterns in the cardiac cavities during ventricular filling and ejection was supposed. This work was intended in order to show that the intraventricular trabeculation plays the determining role in such a flow formation and to find some analytical approaches for its analysis. The morphometric study of human left-ventricular and aortic corrosion casts and dynamic measurement of the aorta by MRI-technique were performed. The data were analysed by means of the "Mathematica" program. Two groups of trabecules were identified that refer to the inlet and outlet of the ventricular blood flow. The first group consists of trabecules of the free left-ventricular wall. The second group consists of long trabecules going along the anterior left-ventricular wall and intracavital lines of the papillary muscles. Both are twisted clockwise and converge in the flow direction. Each group of trabecules is oriented towards the mitral or aortic valve orifices, correspondingly. It was concluded that the helical trabecular organization acts as flow directing paddles that change their mutual orientation during the cardiac cycle evolution. The reorientation of the flow takes place due to sequential contraction of the ventricular structures. The formalization of trabecular orientation will allow one to calculate improved models of implantable substitutes and auxiliary devices for cardio-vascular surgery.

1. Introduction

Flow behaviour in the transport part of the circulatory system is relevant to a number of important points in cardio-vascular physiology and practical medicine. It should be especially taken into account with the design of cardio-vascular substitutes and auxiliary devices such as valvular and vascular prostheses, implantable artificial heart, heart–lung bypasses, and so on. Until now only a few attempts have been made in this field because of the lack of both analytical approaches and precise quantitative descriptions of the flow.

There are several points of view to be taken into account, when dealing with the flow pattern at the transport part of the cardio-vascular system [1–3]. We have decided that the most probable idea is that of the twisted flow pattern in this area. The hypothesis of twisted flow existence in the left ventricle and aorta has been proposed several times, for example, by Rushmer [4], Ohlsson [5], Burakovsky *et al.* [6], Roeva [7], Zakharov *et al.* [8], and others [9–11]. The anatomical properties of the heart and the high energetic and compensatory efficiency indirectly testify in favour of twisted flow patterns.

In conformity with this hypothesis several models of implantable devices for surgical purposes have been designed [7, 12]. However, no information has been found concerning clinical application or experimental evaluation of such devices, which may be explained by the complexity of the problem and the absence of a quantitative model that would allow one to establish a geometry adequate for real conditions.

Therefore, the purpose of this present work was to determine quantitative characteristics of the mutual orientation of intraventricular structures that form the intracardiac and aortic flow, and to try to apply analytical methods for their description. Subsequently the results of this analysis could be used for the calculation and design of improved models of implantable substitutes and auxiliary devices for cardio-vascular surgery.

1.1. Analytical solution of the Navier–Stokes and continuity equations for the class of axis-symmetrical centripetal flows, used in the study

The supposed flow model should take into account the flowing medium viscosity, flow non-stationarity

† Sadly deceased.

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and the possibility of the flow twisting, as was suggested in the model [13]. The analytical solution of Navier–Stokes and continuity equations in terms of velocity may be expressed as the following system:

$$V_r = -C_0(t)r$$
$$V_z = 2C_0(t)z + C_1(t)$$
$$V_\phi = \sum(\gamma_i/2\pi r)\,(1 - e^{-C_0 r^2/2\nu})$$

where

r, z, ϕ = current cylindrical coordinates
V_r, V_z, V_ϕ = radial, longitudinal and tangential velocity components
γ_i = partial velocity circulations
$C_0(t), C_0$ = time-dependent radial velocity gradient and its characteristic value
$C_1(t)$ = time-dependent function, defined by the boundary conditions
ν = kinematic viscosity
t = time.

This model concerns the class of axis-symmetrical flows with radial and longitudinal velocity components that do not depend on medium viscosity, and a twisting velocity component that does depend on medium viscosity. The proposed analytical solution takes into account the pulsatility of the flow, since the function $C_0(t)$ depends on time, and it is suggested that the shape of the flow is confusorial when the value of this function is positive. Thus, the structure of these flows determines their stability and energetic efficacy. This flow can easily divide itself into two or more secondary flows due to proportional division of the circulation member (γ_i).

It was shown that, due to non-stationarity of the flow and rapid change of the longitudinal velocity, the boundary layer that should be formed near the wall is thin enough to neglect its contribution in the cavity volume [14]. On the other hand, one can suppose the existence of another type of boundary layer in flows of this type, since the streamline orientation always supposes that the medium flows around the concave surface. It is well known that this type of flowing around leads to the generation of nearwall Görtler microvortices that provide the fulfilment of condition of flow adhesion to the wall surface and flow sliding upon the microvortices boundary layer. The boundary layer consisting of microvortices should be significantly thinner than the layer forming in the ordinary chaotic flow [15]. Therefore, one can consider that the flow geometry corresponds to the geometrical conditions offered by the cavity in which flow takes place, neglecting the boundary layer thickness.

The analytical solution allows us to calculate trajectories of the flow streamlines. Thus, in the longitudinally-radial projection the product of the longitudinal and squared radial coordinates should be constant:

$$\partial r/V_r = \partial z/V_z; \qquad z_i r_i^2 = \text{Const}$$

where z_i, r_i are current longitudinal and radial coordinates, respectively.

In the tangentially-radial projection the streamlines represent an axis-symmetrical helix, in which twisting

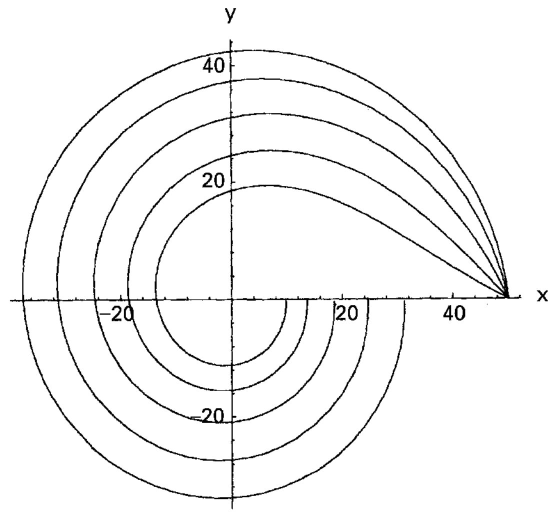


Figure 1 The radially-tangential projection of the calculated twisted flow streamlines. The ratio V_f/V_r is equal to 0.25, 0.5, 1.0, 2.0, 4.0 for the curves from inside to outside correspondingly (Cartesian-coordinate system is scaled in mm).

depends on the relation between tangential and radial velocity components (Fig. 1):

$$\partial r/V_r = r\,\partial\phi/V_\phi$$
$$\phi_i = \phi_0 + [V_\phi(R_0)/2V_r(R_0)](R_0^2/r_1^2 - 1)$$

where ϕ_0, R_0 are the initial values of tangential and radial coordinates respectively and ϕ_i the current tangential coordinate.

The dependence of the longitudinal coordinate on the tangential one should be linear, since both of them are proportional to the same member "$1/r^2$".

According to classical hydrodynamics one should note that the existence of any structures jutting out in the flowing medium will affect the flow, changing the pattern of flow around the wall and therefore in the whole volume of the flowing medium. That is why the mutual orientation of interventricular structures at a given moment of the cardiac cycle play a determining role in instantaneous blood flow behaviour. Admitting that the lines of trabecules and papillary muscles orientation correspond to the intracardiac flow streamlines we have tried to describe them by means of the proposed hydrodynamical flow model.

2. Experimental methods

The study was performed using anatomical corrosion preparations of the left ventricle and aorta obtained from whole corpses within 24 h after death in persons without visible cardiac or vascular pathology. The canine casts were prepared just after euthanasia. The casts were prepared with acrylic blend compounds, and filling was performed under physiologic pressure.

The morphometric studies of the left ventricular casts were performed using an in-house measuring instrument which allowed us to determine the cylindrical coordinates of each point on the cast surface. In doing this, we registered the lines of the trabecules and papillary muscles orientation. The data were processed by means of the "Mathematica" program, which gives a stereometric image of trabecules from several different points of view [16].

The aortic cast dimensions were measured with a caliper in several directions: always the least value of radius was taken into account. The measurement of aortic shape under dynamic conditions was performed using the MRI-technics. The aortic radii measurements were made only on cross-sectional images. The longitudinal coordinates were determined from reference images made along the aorta. The functional approximation of the aortic shape parameters was also made by means of the "Mathemetica" program, applying a least squares method [16].

3. Results

3.1. Classification of the left-ventricular casts according to the cardiac cycle phase*

In order to refer each cast to a specific moment of the cardiac cycle it was necessary to determine which phase of cardiac contraction is reflected in the casts of different geometry. According to the work of Ross *et al.* [17] one should admit that there are considerable signs of the corresponding cardiac contraction phase in each cast. We have tried to observe this difference, provoking cardiac arrest in dogs during the diastole by fast infusion of potassium chloride, and in systole, by means of fast intraventricular infusion of calcium dichloride. The difference in the shape of casts consists of significantly enlarged papillary muscles, different expression of ventricular trabeculation and an evident difference in the left ventricular volume (Fig. 2).

These observations have given us reason to classify prepared human left-ventricular casts according to the cardiac cycle moment, when death has occurred. As a result we have been able to recognize three main types of casts using the ventricular shape, the expression of the papillary muscles contraction and the state of trabeculation as criteria (Fig. 3).

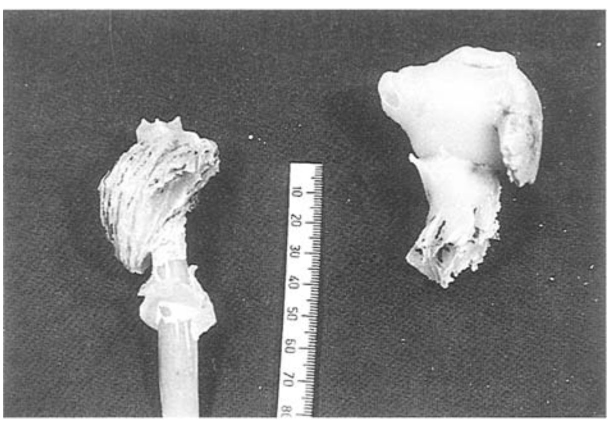


Figure 2 Left-ventricular casts made in dog just after euthanasia. Left: cardiac arrest provoked by fast infusion of a potassium chloride solution; right: cardiac arrest provoked by fast intracardiac infusion of a calcium dichloride solution.

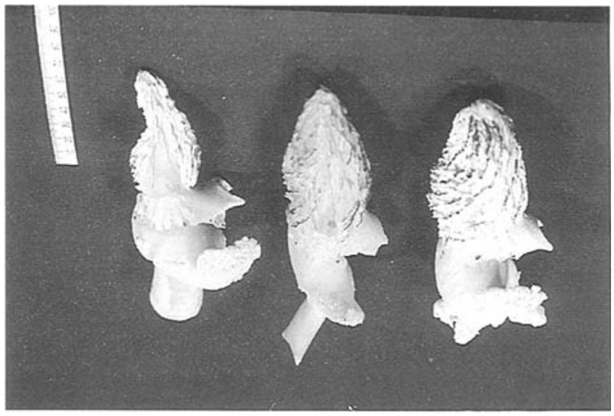


Figure 3 Human left-ventricular cast preparations that were referred to as middle-diastolic (left), systolic (centre) and end-systolic phase (right) of cardiac cycle.

The first type is characterized by obvious helical trabeculation, running from the basic part of the left ventricle to the apex, across the ventricular free wall. The papillary muscles are not enlarged and are concealed among the trabecules. In our opinion this type of cast may be referred to as the middle of diastole.

The second type is recognized by pronounced enlargement of the papillary muscles, with the remaining helical trabeculation at the free ventricular wall. One can also see enlarged vertical trabecules that go along the anterior wall of the left ventricle. This type of left-ventricular cast may be referred to as the beginning of cardiac ejection.

The third type of cast may be referred to as the end of ventricular ejection. These casts have enormously enlarged papillary muscles that almost close the inlet part of the left ventricle. The trabecules that run along the anterior ventricular wall are easily visible.

3.2. Flow-determining structures that form the inlet flow of ventricular filling

In the diastolic cast (our classification) one can see a well-expressed network of free-wall trabecules that form the regular helix, directed from the left-posterior part of the mitral fibrous ring, over the free left ventricular wall to the anterior near-apical zone (Fig. 4a). This helix converges in the apical direction. The vertical trabecules of the anterior wall and papillary muscles being directed towards the aortic valve, also form the converged system attached to the outlet part of the left ventricle.

In the orthogonal projection the free wall trabecules are seen as a regular axis-symmetrical helix oriented from the centre of the mitral valve to the ventricular apex and twisted clockwise in this direction (Fig. 4b). Other intraventricular structures, such as long vertical trabecules of the anterior wall and papillary muscles do not form any obvious regular helix, neither do they cross the trabecules of the free wall.

* This classification is valid in so far as one neglects the afterdeath changes in heart anatomy.

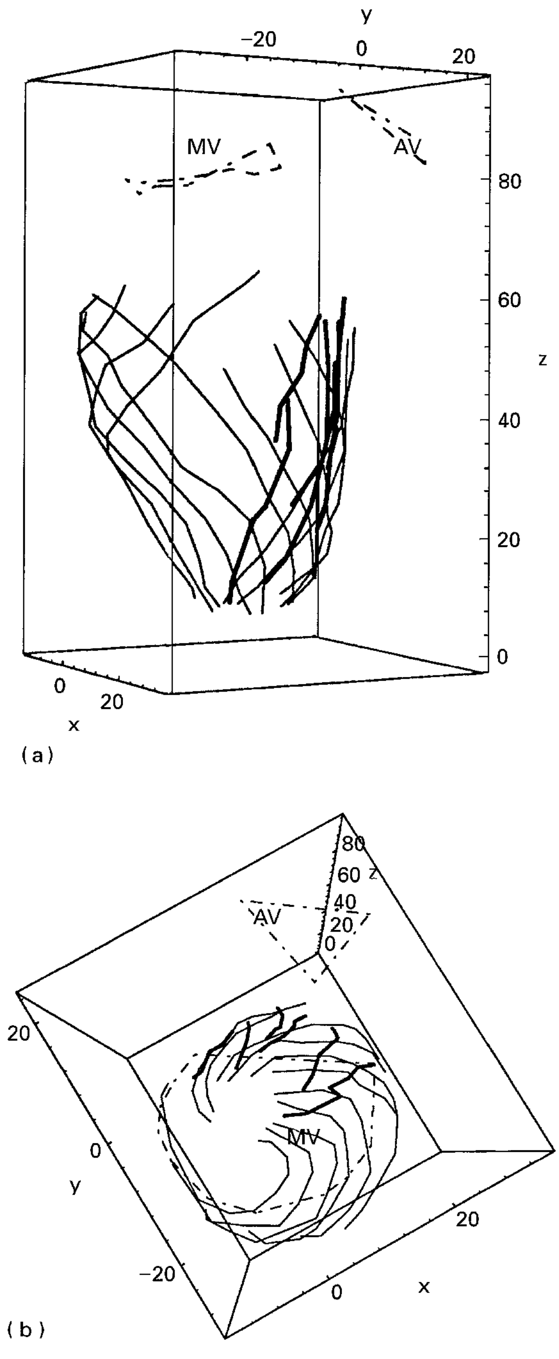


Figure 4 Three-dimensional reconstruction of trabecules and papillary muscles orientation in the human middle-diastolic cast: (a) frontal view from the posterior to anterior left-ventricular wall; (b) transversal view from the apex to the mitral valve (AV – aortic valve, MV – mitral valve, thin lines – free wall trabecules, thick lines – anterior wall trabecules and papillary muscles; Cartesian coordinate system is scaled in mm).

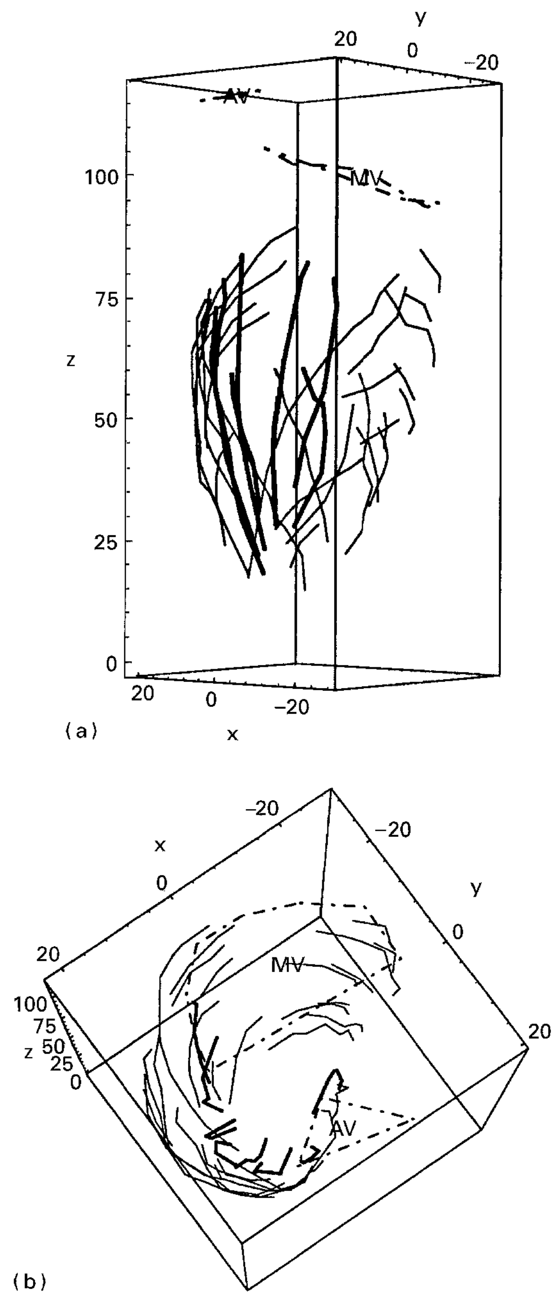


Figure 5 Three-dimensional reconstruction of trabecules and papillary muscles orientation in the human systolic cast: (a) frontal view from the anterior to posterior left-ventricular wall; (b) transversal view from the apex to the aortic valve (AV – aortic valve, MV – mitral valve, thin lines – free wall trabecules, thick lines – anterior wall trabecules and papillary muscles; Cartesian coordinate system is scaled in mm).

3.3. Flow-determining structures that form the outlet flow during ventricular ejection

In the cast that corresponds to the beginning of systole one can see the remaining systemic orientation of the free-wall trabecules even though its symmetry is already distorted (Fig. 5a). In this cast the system of vertical trabecules of the anterior wall and papillary muscles form a significantly more regular group. The mutual convergence of these lines is easily seen in the upper part. One should note that these lines are situated in the centre of the ventricular cavity, their principal orientation remaining towards the aortic valve, therefore crossing the free-wall trabecules.

Changing the view point, it is possible to find a cast position in which the papillary muscles and the long vertical trabecules of the anterior wall form a pronounced alternative helix oriented from the ventricular apex to the centre of the aortic valve and twisted also anticlockwise in this direction (Fig. 5b). This helix is less inclined than the inlet one. At this phase the system of free-wall trabecules is concealed by the alternative system of helically organized vertical trabecules and papillary muscles that should be exposed to the blood flow at the first turn.

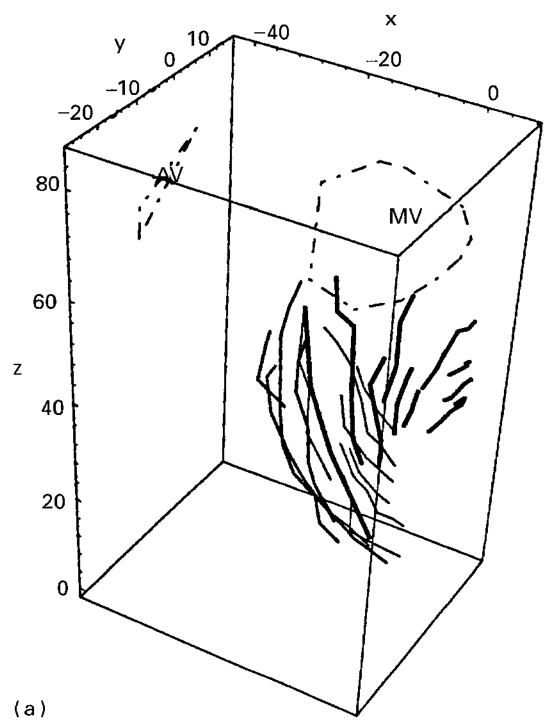
3.4. Cast shape at the end-systole

In the end-systolic casts almost all the volume is taken up by contracted papillary muscles. Common orientation of the intraventricular structures in these casts is attached, primarily, to the inlet part of ventricle: all the trabecules and papillary muscles are directed together from the mitral valve to the ventricular apex (Fig. 6a). They form a system of lines of equal orientation converging towards the ventricular apex.

In the orthogonal projection this type of trabecular orientation demonstrates that the system is ready to accept the inlet blood flow from the mitral orifice since it is twisted in the same direction as in the case of the first middle-diastolic cast-type (Fig. 6b). The trabecules and the papillary muscles turn in the same direction, forming a common helix.

Therefore, inlet and outlet flow-determining structures seem to affect the flow medium in a consequent manner, that is, at the phase of ventricular filling the free-wall trabecules play the main role in flow organization, whereas at the ejection phase they are concealed by contracted papillary muscles and anterior-wall vertical trabecules. Before the filling phase begins, all the intraventricular structures are oriented in such a way that they are ready to accept the inlet blood flow from the left atrium.

Thus, on the base of casts analysis, the quantitative characteristic of mutual orientation of anatomical intracardiac structures was obtained. Keeping in mind the fact that at different phases of cardiac contraction these structures are moved around by the blood flow, they should play a determining role in the instantaneous flow pattern, that is, the streamlines of flow should coincide with the instant lines of trabecules or papillary muscles at the corresponding moment of the cardiac cycle.



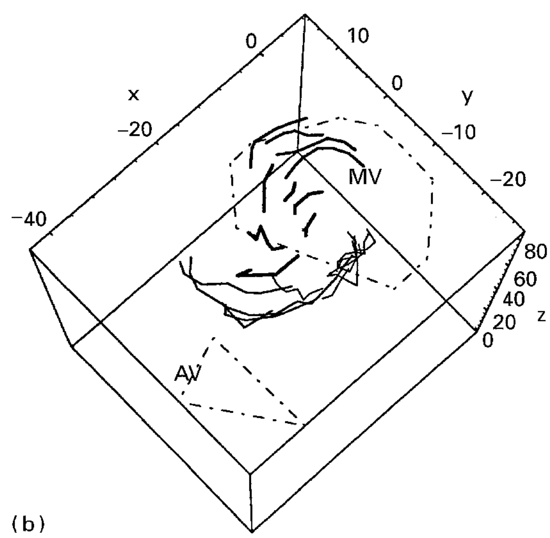


Figure 6 Three-dimensional reconstruction of trabecules and papillary muscles orientation in the human end-systolic cast: (a) frontal view from the anterior to posterior left-ventricular wall; (b) transversal view from the apex to the mitral valve (AV – aortic valve, MV – mitral valve, thin lines – free wall trabecules, thick lines – anterior wall trabecules and papillary muscles; Cartesian coordinate system is scaled in mm).

3.5. Comparison of real and theoretical geometrical correlations in the aorta and the left ventricle

In order to confirm that the relation "$z_i r_i^2 = \text{Const}$" is fulfilled, we analysed the geometry of the aorta, since the flow in the aorta is fully developed, and therefore should correspond to the initial flow, generated in the ventricular cavity. It was discovered that in the aorta the relation between $\boldsymbol{R}$ and $\boldsymbol{Z}$ is fulfilled to a very high level of precision (Fig. 7). The most pronounced

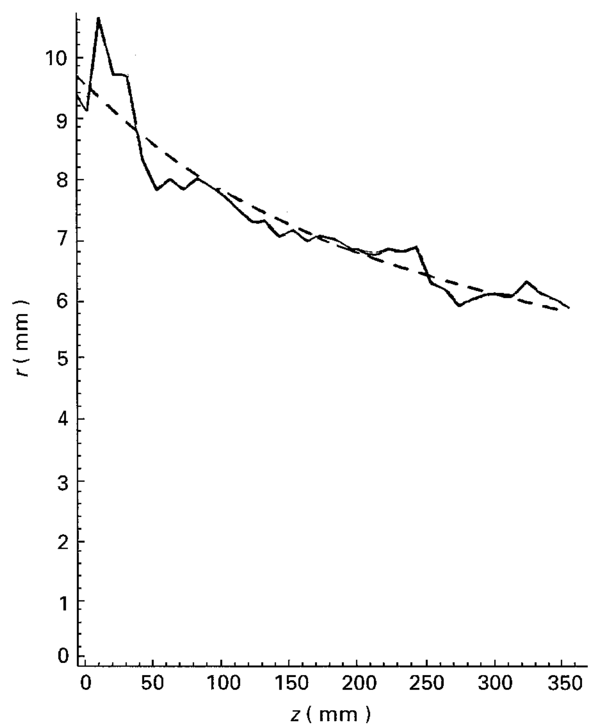


Figure 7 The human aortic shape reconstruction in radially-longitudinal terms and its approximation to the function $z(r) = C/r^2 - Z_0$, where C – constant, Z_0 – calculated initial value of the longitudinal coordinate ($C = 17\,645.6, Z_0 = 183.291$ mm, aortic cast measurement, scale in mm).

deviation takes place only at the points of the large branches, that is, the brachiocephalic and splanchnic vessels ramification.

The same results were obtained by measuring the dynamic geometrical correlations in the human aorta by means of MR-imaging (Fig. 8). It is seen that the dependence "$z_i r_i^2$ = Const" is fulfilled almost at each moment of the cardiac cycle.

The theoretical streamlines at the radially-tangential projection are seen as a symmetrical helix, in which twisting depends on the relation between tangential and radial velocity components (Fig. 1). One can see that in the "Z–Φ" projection the lines of trabecular orientation form an almost ideal helix that is similar to the theoretical streamline graphics (Fig. 9a, b). It was calculated that the V_Φ/V_r-relation in the heart changes in the limits of 0.25 to 2.

The trabecule lines reconstruction in "Z–Φ" coordinates shows that most of the trabecular length fulfils the condition of linearity (Fig. 10).

4. Discussion

Close coincidence between the calculated theoretical streamlines and the real lines of the intracardiac structures orientation was observed. Considering that these lines should correspond to real intracardiac and aortic flow streamlines it becomes possible to perform

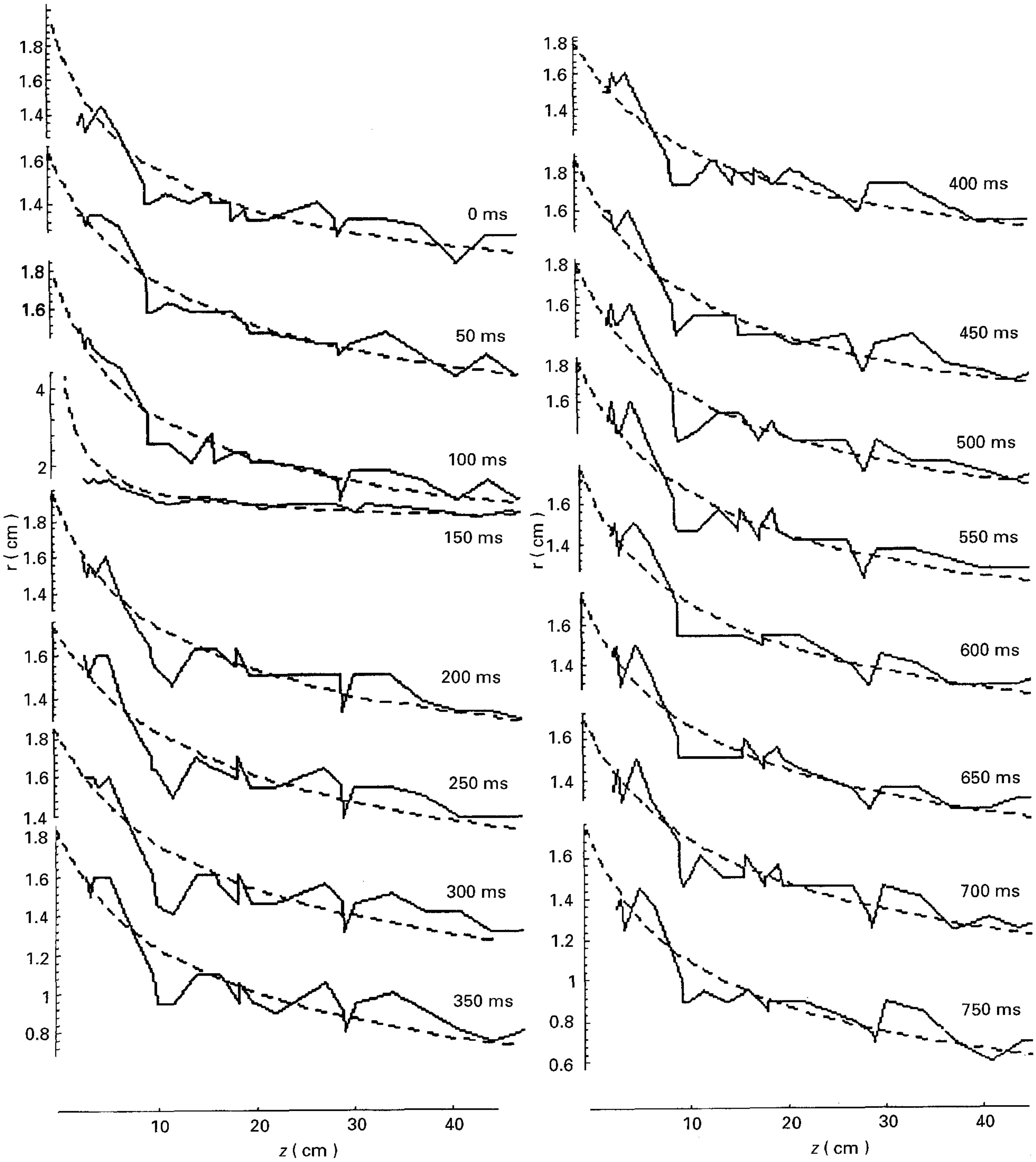


Figure 8 The dynamic MRI-measurement of aortic radially-longitudinal relations in the course of cardiac cycle evolution (the delay after R-ECG is noted near each curve) and corresponding approximating curves (scale in cm).

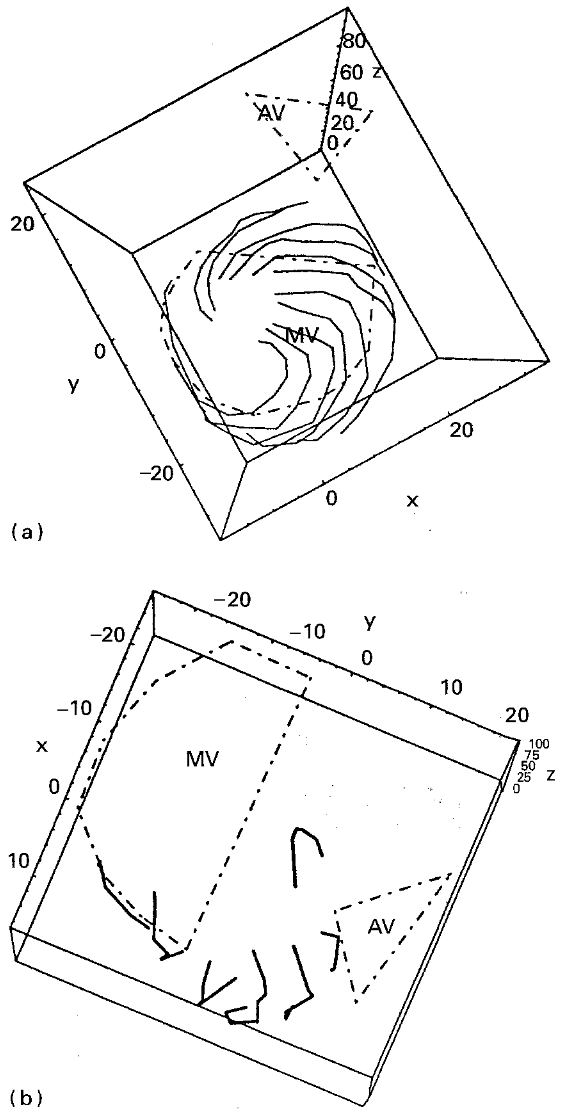


Figure 9 Three-dimensional reconstruction of diastolic trabecular lines in the middle-diastolic human left-ventricular cast (a) (transversal view from the apex to the mitral valve); and systolic trabecular and papillary muscle lines in the systolic human left-ventricular cast (b) (transversal view from the apex to the aortic valve).

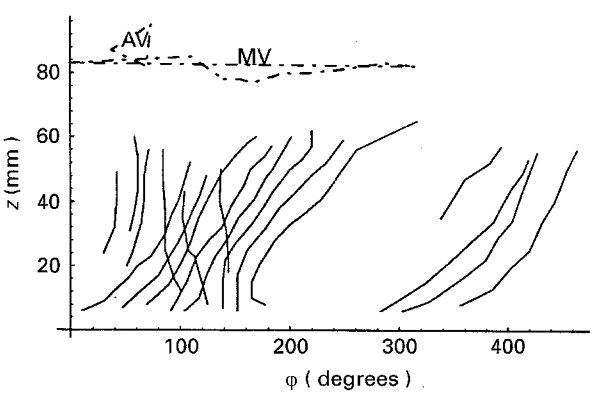


Figure 10 Trabecular and papillary muscles lines reconstruction in tangentially-longitudinal coordinates in the middle-diastolic human left-ventricular cast: the linearity fulfilment (AV – aortic valve, MV – mitral valve, Cartesian coordinate system is scaled in mm).

detailed analysis and to almost reconstruct the real blood flow in the transport part of the circulatory system.

In [13] a detailed analysis of the twisted tornado-like flows was undertaken. In particular, some of the properties of such a flow were mentioned, that would be important in the application of the proposed model to the cardio-vascular system.

Flows of this type dissipate energy only in the narrow axial zone in which the radius is smaller than $\sqrt{2\nu/C_0}$. It is well known that arterial pressure is kept virtually constant throughout all the large and medium arteries. This means that the energy dissipation in flowing blood is practically non-existent at the level of large arterial vessels. Indeed it was shown that heat shunting in the dog coronary vascular tree takes place only at an arteriolar level [18].

It was calculated that the transversal pressure distribution across the twisted flow has its minimum in the axial zone, while the pressure near the wall is maximal [13]. The extent of the central depression zone is proportional to azimuthal velocity squared, i.e. a higher degree of twisting leads to more flow self-suction to some basic substrate, and/or provides additional longitudinal and radial flowing medium transportation. From this point of view the twisting flow modulation in the left ventricle plays a very important role in left atrium draining and mitral valvular mechanics. The maintenance of flow twisting during ventricular ejection should also determine the function of the aortic valve.

It also appears that the transversal transport of substances out of the flow lumen is totally impossible except by diffusion, because there are no streamlines leading out of the lumen. In fact, in large and medium arteries, the transversal transport of substances outward does not take place, and it is the autonomic vascular system, named *vasa vasorum*, that provides artery wall nutrition. It was confirmed that the transversal transport in large and medium arteries attains no more than 1/3 of the vascular media thickness [19], which may be explained by diffusion.

There has been no satisfactory explanation to account for the well known phenomenon of the blood flow boundary layer which is free of cellular elements (for example, [20]). This phenomenon may be explained by flow separation under the influence of the inertia forces of twisting flow due to the density difference between blood cells and plasma. This layer analysis is particularly important since the boundary interactions in flows of this type seem to be arranged differently to those in other flow-types.

The energy which a well developed twisted flow transfers longitudinally, is maximal when the azimuthal and longitudinal flow velocity components are equal [13]. Based on this fact and on the analytical solutions one concludes that as a result of twisting these flows are very stable, since they can accumulate a sufficient amount of energy as a fly-wheel. This is why the time of flow initiation may be very short, whereas the time of its extinction is essentially longer and the flow may retain its properties even when the conditions that have caused flow initiation have already changed.

One must admit that the cardio-vascular system is a self-modelling system. Thus in the absence of stagnation zones, its lumen shape coincides with the shape of the flow pattern and changes during the cardiac cycle following flow evolution. So it is reasonable to assume that the relations between the luminal geometry and the luminal flow are optimized towards minimum energy loss, and that the luminal geometry is able to change corresponding to variations in flow conditions (for example in the presence of pathological disorders) in order to minimize the energy dissipation. Thus, Roeva [7] has shown the phenomenon of cardiac trabecules reorientation following the creation of a new outlet from the left ventricle through the left ventricular apex.

Apart from the spiral trabecular orientation, one may suppose that other flow-twisting modulation mechanisms exist. For example, all large and medium arterial vessels contain spirally-oriented elastic fibres in their walls [8]. Thus, due to radial expansion, the vessel itself moves around its axis as the pulse wave propagates. During systolic contraction the heart also rotates around its axis [21]. This rotational movement may take part in the flow twisting modulation.

The described features of the cardio-vascular anatomy which are involved in the corresponding physical phenomenon are of fundamental importance to physiology and practical medicine. Thus efforts to maintain the physiological flow may be applied to cardio-vascular prothesizing, heart–lung bypass procedures, diagnostics, etc. Use of the twisting flow model may also be useful in making more accurate some of the concepts in cardio-vascular physiology, such as energetic correlations, valvular mechanics, mechanisms of regulation, and so on.

5. Conclusions and perspectives

The coincidence of the theoretically expected streamlines and the measured geometrical correlations in the left ventricle and aorta confirms the probability of a twisted flow presence in this part of the circulatory system, or at least at the largest part of its volume. This means that the proposed analytical approach is valid and may be used for the design of certain artificial substitutes and devices for corresponding segments of the cardio-vascular system.

The study will be continued along the following principal directions:

1. Blood flow reconstruction on the basis of experimental measurements of the static and dynamic geometrical correlations in the left ventricle and the aorta will be attempted.
2. Analysis of the flow generation in the aortic branches and determination of the twisted flow extent in the arterial system will be carried out.
3. Twisted-flow participation in valvular mechanics will be investigated.

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